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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

18. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Decompose into the sum of two squares the number $17^3 \cdot 73^5$.

I. Solution by the PROPOSER.

$$(a^2 + b^2)(a^2 + b^2) = (a^2 \pm b^2)^2 + (ab \mp ab)^2 = A^2 + B^2.$$

$$(a^2 + b^2)^3 = (A^2 + B^2)(a^2 + b^2) = (Aa \pm Bb)^2 + (Ab \mp Ba)^2.$$

$$\therefore (a^2 + b^2)^3 = \{ a(a^2 - 3b^2) \}^2 + \{ b(3a^2 - b^2) \}^2 \\ = \{ a(a^2 + b^2) \}^2 + \{ b(a^2 + b^2) \}^2.$$

$$\text{Similarly } (c^2 + d^2)^5 = \{ c(c^4 - 10c^2d^2 + 5d^4) \}^2 + \{ d(5c^4 - 10c^2d^2 + d^4) \}^2 \\ = \{ c(c^2 + d^2)^2 \}^2 + \{ d(c^2 + d^2)^2 \}^2 \\ = \{ c(c^2 + d^2)(3d^2 - c^2) \}^2 + \{ d(c^2 + d^2)(3c^2 - d^2) \}^2.$$

$$\text{Let } a=4, b=1, c=8, d=3. \therefore 17^3 = 52^2 + 47^2 = 68^2 + 17^2.$$

$$73^5 = 10072^2 + 44403^2 = 42632^2 + 15987^2 = 21608^2 + 40077^2.$$

$$\therefore 17^3 \cdot 73^5 = 3092572^2 + 788035^2 = 2357900^2 + 2150653^2 \\ = 362372^2 + 3170755^2 = 1811860^2 + 2627197^2 \\ = 3190628^2 + 69955^2 = 2848180^2 + 1439747^2 \\ = 3099580^2 + 560003^2 = 1068428^2 + 3007235^2 \\ = 2835028^2 + 1465475^2 = 1172380^2 + 2968253^2 \\ = 2782340^2 + 1565197^2 = 1835572^2 + 2612685^2.$$

\therefore the sum of two squares in twelve ways.

II. Solution by H. W. DRAUGHON, Olio, Mississippi and O. W. ANTHONY, Missouri Military Academy, Mexico, Missouri.

$$\text{Since } 17^3 \times 73^5 = 17^2 \times 73^4 \times 17 \times 73 = 17^2 \times 73^4 \times 1241.$$

$$\text{And } 1241 = 35^2 + 4^2 = 29^2 + 20^2. \text{ Therefore,}$$

$$17^3 \times 73^5 = (17 \times 73^2 \times 35)^2 + (17 \times 73^2 \times 4)^2 = (17 \times 73^2 \times 29)^2 + (17 \times 73^2 \times 20)^2.$$

III. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Solution by determinants,

$$17 = \begin{vmatrix} 4 & -1 \\ 1 & 4 \end{vmatrix}, \quad 73 = \begin{vmatrix} 8 & -3 \\ 3 & 8 \end{vmatrix}, \quad 17^2 = \begin{vmatrix} 17 & 0 \\ 0 & 17 \end{vmatrix}, \quad 73^4 = \begin{vmatrix} 5329 & 0 \\ 0 & 5329 \end{vmatrix}$$

$$17^3 = \begin{vmatrix} 17 & 0 \\ 0 & 17 \end{vmatrix} \times \begin{vmatrix} 4 & -1 \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} 68 & 17 \\ -17 & 68 \end{vmatrix}$$

$$17^3 \times 73 = \begin{vmatrix} 68 & 17 \\ -17 & 68 \end{vmatrix} \times \begin{vmatrix} 8 & -3 \\ 3 & 8 \end{vmatrix} = \begin{vmatrix} 344-51 & 204+136 \\ -136-204 & -51+344 \end{vmatrix} = \begin{vmatrix} 493 & 340 \\ -340 & 493 \end{vmatrix}.$$

$$\text{Then } 17^3 \times 73^3 = \begin{vmatrix} 493 & 340 \\ -340 & 493 \end{vmatrix} \times \begin{vmatrix} 5329 & 0 \\ 0 & 5329 \end{vmatrix} = (493^2 + 340^2)(5329)^2 \\ = (73)^2(493)^2 + (73)^2(340)^2, \text{ which is one set of answers.}$$

Also solved by *R. J. Adcock, and M. A. Gruber.*

19. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Find three positive integer numbers whose sum is a cube, and, also, the sum of any two diminished by the third a cube.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x , y , and z = the three positive integers.

$$\begin{aligned} \text{Then } x+y+z &= a^3 \\ x+y-z &= b^3 \\ x+z-y &= c^3 \\ y+z-x &= d^3 \end{aligned}$$

$$\text{Whence } x+y+z = b^3 + c^3 + d^3 = a^3;$$

$$x = \frac{b^3 + c^3}{2}; y = \frac{b^3 + d^3}{2}; z = \frac{c^3 + d^3}{2}.$$

This is a problem in which the sum of three cubes = a cube. Take $3^3 + 4^3 + 5^3 = 6^3$. But as the numbers are to be integers, we multiply by 2,

and obtain $6^3 + 8^3 + 10^3 = 12^3$. $\therefore x = \frac{6^3 + 8^3}{2} = 364$; $y = \frac{1}{2}(6^3 + 10^3) = 608$; and

$z = \frac{1}{2}(8^3 + 10^3) = 756$. The number of answers is infinite.

II. Solution by R. J. ADCOCK, Larchland, Warren County, Illinois.

Let x , y , z , be the three numbers; then $x+y+z=u^3$, $x+y-z=r^3$, $x+z-y=s^3$, $z+y-x=s^3$, by conditions.

$$\text{Wherefore } x+y+z=u^3=x^3+r^3+s^3, \quad x=\frac{r^3+r^3}{2}, \quad y=\frac{1}{2}(r^3+s^3),$$

$z=\frac{1}{2}(r^3+s^3)$. The most general equation yet obtained by me for the sum of three cubes = a cube, is found from,

$[(ax^3+dy^3)x]^3 + [(bx^3+hy^3)y]^3 + [(cx^3-hy^3)y]^3 = [(ax^3+gy^3)x]^3$, by expanding, equating coefficients of similar terms with respect to x and y , eliminating d , g , and h , giving the identical equation,

$$[9a^3bx^3y + (b^2-bc+c^2)^2y^4]^3 + [9a^3cx^3y - (b^2-bc+c^2)^2y^4] \\ + [9a^4x^4 - 3(b^2-bc+c^2)caxy^3]^3 = [9a^4x^3 + 3ab(b^2-bc+c^2)xy^3]^3.$$

By numbers for the letters in the above, some of the resulting